# Revisiting Yagi Element Scaling

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For a long time I have been interested in Yagi antennas. The Yagi, or more technically correct, the Yagi-Uda antenna was first described in 1926 in a paper written by Shintaro Uda and Hidetsugu Yagi (1). The first English language reference was published in 1928. I'll not go into the history of the Yagi or specifics of the design properties of the Yagi given the amount of material that has been publish over the years. There are numerous references that I have included at the end of this paper if you want to find out more about how they work, how to optimize them, and other research specific to the performance properties of the Yagi's. The intent of this paper is to focus on one of the more misunderstood properties of the Yagi that is often overlooked, element scaling.

What do I mean by "*element scaling?*" One of the challenges an antenna designer faces is scaling. This happen when you want to take an antenna designed for a given frequency and scale it to a different frequency. Programs like EZ NEC and others have a scaling feature but there is one problem, they will often scale the element diameter to a size that may not be mechanically strong enough for use in the real world or it may scale the elements to a size that is not manufactured or readily available. By allowing the designer to select the element diameter as a part of the scaling inputs the designer can design an antenna that is ideally suited for the environment it will exist in and with materials that are readily available.

Let me cite an example. Let's say we have a design for a 2 meter Yagi that uses small diameter .2" aluminum rod. If we use EZ NEC to scale the antenna down to 432 MHz we end up with extremely small elements that are unrealistic to use. When we go back and resize the elements in our model we find that the properties for the 432 MHz model are no long electrically ideal and we end up having to redesign the antenna and manually scaling the elements.

The solution is to allow the designer to specify the element diameter for the target antenna and then let the software properly size the elements lengths so that our 432 MHz model has approximately the same electrical characteristics as our 144 MHz model and is using element diameters we specified. This is the problem we are attempting to solve with theses formulas.

## Radius Scaling: Giving Credit where credit is due

In 1980 Dr. James Lawson publish several articles in "Ham Radio" about the Yagi antenna beginning with the January issue (3,4,5). The specific area of focus of this paper is based on his article published in the December issue (5). Dr. Lawson later compiled his articles into a book titled: "*Yagi Antenna Design*" publish by the ARRL in 1986. It is one of the most widely cited books on Yagi design. Much of the content of this paper is taken directly from his article published in December 1980 and from supplemented by material found in chapter 7, pages 7.3-7.5 of his book (6). I must give credit where credit is due, I used much of his original text, formulas and examples. That's why this article entitled "Revisiting" Yagi Element Scaling. We will be revisiting his work. My intention was to leverage his approach but to restate the formulas using a form consistent with many of today's programming languages and spreadsheet programs. I also added some additional text to help explain some of the areas that I struggled with.  I hope this will make it a bit easier to understand his process and to allow the reader to use his excellent formulas in designing and scaling their own Yagi designs.

Below is an example of how I converted some of his formulas.  The equation on the left is how it would be stated by Dr. Lawson, the equation on the right is how it would be entered as a spreadsheet formula.

$b= 5 \log y^{(-2)}$ = 5 * log(y)^-2

Also note the formulas are specific to elements with a consistent diameter, elements that are cylindrical rather than tapered.  There is another set of formulas to use when scaling tapered elements which we will not cover in this article.  It's my thought that before tackling the formulas for scaling tapered elements one should master the formulas for cylindrical elements.

## Radius Scaling: Introduction

Any Yagi antenna design, such as those shown in Table 1, can be scaled either to other frequencies or to elements of different diameter at the same center frequency.  Since all design parameters are dimensions expressed in wavelengths at a central design frequency the values shown in Table 1 are invariant to frequency scaling and therefore the behavior of the antenna will be unaffected by the choice of central design frequency.  However this is true only if all physical dimensions (including element radius) are adjusted in proportion to the desired wavelength.

Experience shows that practical element radii expressed in wavelengths are not constant; at low frequencies (long wavelengths) relatively thin elements are used while at high frequencies relatively thick elements are typical.  How then can a given design be altered to an equivalent design where element radius is changed?  The clue is to make the impedance of the changed element exactly the same as the impedance of the original element; in this way exactly the same element currents will flow, resulting in the same overall antenna performance.

| | 2 Element Yagi | | 3 Element Yagi | | 4 Element Yagi | | 5 Element Yagi | | 6 Element Yagi | |
|---|---|---|---|---|---|---|---|---|---|---|
| element | X | LE | X | LE | X | LE | X | LE | X | LE |
| R | 0.000 | 0.49366 | 0.000 | 0.49801 | 0.000 | 0.49185 | 0.0000 | 0.49994 | 0.000 | 0.49528 |
| DR | 0.150 | 0.47050 | 0.150 | 0.48963 | 0.250 | 0.47900 | 0.1875 | 0.48040 | 0.150 | 0.48028 |
| D1 | | | 0.300 | 0.46900 | 0.500 | 0.46319 | 0.3750 | 0.45232 | 0.300 | 0.44811 |
| D2 | | | | | 0.750 | 0.46319 | 0.5625 | 0.45232 | 0.450 | 0.44811 |
| D3 | | | | | | | 0.7500 | 0.45232 | 0.600 | 0.44811 |
| D4 | | | | | | | | | 0.750 | 0.44811 |
| Gain (dBi) | 6.88 | | 7.86 | | 10.62 | | 10.45 | | 10.7 | |
| F/B | 7.94 | | 23.6 | | 41.62 | | 32.27 | | 52.71 | |
| Preferred Yagi antenna designs.  All elements have radius RO, of .0005260 λ and boom position X in λ | | | | | | | | | | |

*Table 1: Preferred Yagi Designs*

| | Reflector | | | Driven Element | | | Director | | |
|---|---|---|---|---|---|---|---|---|---|
| | Case 1 | Case 2 | Case 3 | Case 1 | Case 2 | Case 3 | Case 1 | Case 2 | Case 3 |
| LE | 0.49528 | 0.49489 | 0.49465 | 0.48028 | 0.47876 | 0.47785 | 0.44811 | 0.44431 | 0.44204 |
| FR | 0.97252 | 0.97042 | 0.96917 | 1.00289 | 1.00311 | 1.00325 | 1.07489 | 1.08090 | 1.08451 |
| X (ohms) | 30.40800 | 30.40800 | 30.40080 | -3.14700 | -3.14700 | -3.14700 | -78.58200 | -78.85200 | -78.85200 |
| Case 1: Ro=.0005260 λ, Case 2: Ro=.0008 λ, Case 3: Ro=.0010 λ | | | | | | | | | |

*Table 2: Scaled Antennas*

## Setting Up the Scaling Parameters

Before we get into the math we need to take a moment and explain an idea that you will see expressed in several places, the idea of describing antenna dimensions based on wavelengths rather than meters, feet or inches. You are going to see *F=1* used a lot, or $X_{(F=1)}$. To understand the *F=1* idea a little better let's assume that our base design is 14.2 MHz. All measurements are based on the wavelength of 14.2 MHz which is 69.265 feet (831 inches). Expressing dimensions in wavelengths is the first step in scaling an antenna. As an example in, Table 1 the reflector for a two element Yagi is .49366 wavelengths long, which if we were to express that in feet would be 34.193 feet long. So when we say *F=1* we are expressing *F* in wavelengths, thus *F* = one wavelength which at 14.2 MHz is 831 inches. This allows us to scale our dimensions regardless of frequency.

By expressing dimensions in wavelength we can take an antenna designed for 14.2 MHz and scale it for use at 28.3 MHz. As mentioned earlier where this fails is in scaling the element diameters. All the math that follows is all about taking element diameter into account so that you can scale an antenna using realistic element diameters thus allowing you to build an antenna for 10 meters that has the same performance properties as the antenna you modeled from. This is especially handy for scaling antennas for use in the VHF and UHF spectrum.

Expressing antenna relationships in terms of wavelengths also has one other important advantage. It lets you compare element relationships and/or compare one antenna to another. For example seeing a spacing of say 15 inches is not very helpful, but when expressed in wavelengths such as .2 wavelengths it becomes much easier to see the interrelationship of the elements in the antenna as well as being able to make direct comparisons with other similar antennas. Now let's get to our formulas.

Since the (radiation) resistance of the element is essentially unaffected by changes in radius, we need only make the reactance invariant to scaling element radius. Element reactance, *X*, near the resonance can be expressed as:

$$X = R*Q*(F/FR-FR/F) \quad \textbf{(1)}$$

See D11 in attached spreadsheet

Where:

$R$ = the radiation resistance
$Q$ = the effective Q
$F$ = the central design frequency

*FR* = the element resonant frequency, also referred to central design frequency

*RQ* can be accurately expressed as:

$$RQ = (215.15*\log (K) -160) \quad \textbf{(2)}$$

See D14 in attached spreadsheet

Where:

$K = 1/RO$

*RO* = the radius of the element expressed in wavelengths at $F=1$, the central design frequency.

By using equations 1 and 2 we can now derive the element's reactance, *X*.

$$X = (215.15 * \log (K) - 160) * (F/FR - FR/F) \quad \textbf{(3)}$$

See D11 in the attached spreadsheet

And at the central design frequency ($F=1$):

$$X = (215.15 * \log (k) - 160) * (1 / FR - FR) \quad \textbf{(4)}$$

See D11 in the attached spreadsheet

If you want to scale the element radius from an original value we must ensure that $X_{(F=1)}$ is unchanged. Note that $X_{(F=1)}$ contains two variables, (*K* and *FR*), which are a function of element radius *RO*. *FR* is calculated from the physical length of element *LE* and physical resonant length *LER*; both of these lengths are measured in wavelengths, $\lambda_0$, at $F=1$.

$$FR = LER/LE \quad \textbf{(5)}$$

See D13 in the attached spreadsheet

Empirically (see Table 2, case 2 where *RO*=.0008):

$$LER = [1 - (10.7575 * \log (K) - 8) \wedge -1] /2 \quad \textbf{(6)}$$

See D12 in the attached spreadsheet

Thus from equations 5 and 6:

$$FR = (1 - (10.7575 * \log (K)-8) \wedge -1) / (2*LE) \quad \textbf{(7)}$$

## Let's Scale an Antenna

Now that we have establish the basic tools for converting a given antenna such as the one describe in Table 2 let's scale it to a different frequency using appropriately sized elements for the new antenna. An example will illustrate the nature of results. Consider the antenna design for the three-element antenna of Table 1; this would be a reasonable design for a 14.2 MHz antenna where one wavelength is 831.76 inches and elements with a radius of 0.00052599 wavelengths which corresponds to an element diameter of ≈ 7/8 inches.

This would be a reasonable dimension for a mechanically adequate element. Now suppose that we would like an equivalent antenna for 28 MHz where we would want an element radius much larger in terms of wavelengths. Table 2**Error! Reference source not found.** shows the computations. The original design shown as case 1, and case 2 is the scaled design that uses elements with a radius of 0.0008 wavelengths (about 5/8 inches diameter). Case 3 is the same antenna but scaled to use one inch diameter tubing (0.0012 wavelengths radius). Note that the changed values for element lengths are not wholly intuitive because two things happen simultaneously. As the radius increases the Q drops, requiring a greater spread in the resonant frequencies of reflector and director. However, at the same time the resonant physical length also changes.

It is important to reiterate that it is conceptually wrong to scale boom length and element lengths (for example, to convert a VHF antenna design to HF) without also scaling the element radius. The correct way to adjust an antenna element when the old and new radii (in wavelengths) are different is to modify both the element lengths and radius to give the same electrical reactance as the source antenna.

We now have the tools to convert a given antenna design such as shown in Table 2, case 1, to a new antenna design where the element radius is changed; the new antenna will perform exactly the same as the original antenna at the central design frequency. However the frequency-swept behavior of the new antenna, while qualitatively similar to the original, will show a broader or narrower bandwidth depending on the change in element Q.

Let's begin by scaling the three element 14.2 MHz antenna describe in Table 2, case 1, to a new frequency of 28 MHz. The procedure is fairly straight forward. One final clarification, subscript 1 is referencing our 14 MHz (source) antenna, example: $K_1$ is referencing the *K* value of our source antenna. Subscript 2 is referencing our target antenna, example $K_2$. Let's begin by calculating the new scaled element length, $LE_2$:

$$K_1 = 1/RO_1 \; ; \; K_2 = 1/RO_2 \qquad \textbf{(8)}$$

See D16 & D17 in the attached spreadsheet

$$FR_1 = (1 - (10.7575 * \log(K_1) - 8)^{\wedge}\text{-}1)/(2*LE_1) \qquad \textbf{(9)}$$

See D18 in the attached spreadsheet

$$X_1 = (215.15 * \log(K_1)-160) * (1/FR_1-FR_1) \qquad \textbf{(10)}$$

See D19 in the attached spreadsheet

Having calculated reactance at $F=1$, we can now calculate the value of $FR_2$ that will give us the same value of *X* with the new element radius, $RO_2$:

$$A \equiv^{1} X_1 / (215.15 * \log(K_2) - 160) \qquad \textbf{(11)}$$

See D20 in the attached spreadsheet≡

$$FR_2 = (-A + (A^{\wedge}2+4))^{\wedge}.5) /2 \qquad \textbf{(12)}$$

See D21 in attached spreadsheet

[1] The three bar equal sign, ≡, is a symbol that indicates equivalence of two different things in mathematics and logic. It has the appearance of an equals sign with a third line.

$$LE_2 = (1-(10.7575 * \log (K_2) - 8) \wedge -1) / (2*FR_2) \quad \textbf{(13)}$$

See D22 in the attached spreadsheet

Now that we have set up the initial equations we can walk through the process for scaling an antenna. As per Dr. Lawson's suggestion I set up his formulas in a spreadsheet and then tested the results and scaled several antennas and then modeled them using EZ NEC. The results were has predicted. I was able to scale a 2 meter Yagi to 70 cm and the performance numbers and impedances were similar to the 2 meter antenna.

## Using the spreadsheet to scale your own antenna

Dr. Lawson also provided two examples based on the three element antenna described in Table 2, case 1. His results are in Table 2, cases 2 and 3 for the different antennas he scaled. The way you can use this spreadsheet to model you own antenna is to use a program like EZ-NEC(8) and find an antenna that you would like to scale. There are several examples in the ARRL antenna handbook. After you have model the antenna in your antenna design program and are satisfied with the results change the units to wavelengths. Then using that data you can plug the numbers for each element into the table found in the scaling tab of the worksheet.

We should note that some antenna design programs have a "Scaling" feature. As was stated earlier this does not necessary work given that it scales everything, including the element diameters. What you might end up are elements with unrealistic diameters, or material sizes that are not manufactured.

Start with source antenna and enter in the reflector information:

- ✓ Enter the element length into cell D4
- ✓ Enter the element radius in D5
- ✓ Enter the design frequency into D6

Now we are going to enter in the information for the target antenna:

- ✓ Enter the target frequency into D7
- ✓ Select the desired element diameter in D9

**Please note:** In the spreadsheet you have to select the desired element diameter for the target antenna from a drop down list that includes popular tubing diameters. Consequently the element diameters may vary slightly from Dr. Lawson's results. For example in case 2 the element radius is .0008, if you select .75" from the dropdown the element radius will actually be .000889615. This will cause the length to be slightly different than the result in Table 2, case 2. Using .0008 the result is a reflector that is .49489 $\lambda$ long. Using .000889615 the results will be 0.49478 $\lambda$ long, which is a difference of only .11.

The reflector length will be displayed in D26. If you enter that value in cell K6 the program will generate a wires schedule that you can use in EZ NEC. The spacing information can be found in Table 1 for the 3 element Yagi.

After you have completed entering the information for the reflector continue by entering the length for the driven element and then the director. Remember to enter all dimensions in wavelengths. You can convert them in EZ NEC by changing the unit's option to the desired units: meters, feet, inches, etc.

## References:


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